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Lu Tian, PhD

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# VDSP Statistics Manual

## A Guide to the Standardization of 25(OH)D Data Measured in the Past

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Lu Tian, PhD

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# Overview

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# Introduction: Task

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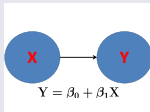
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## Task



*Given 25(OH)D measurements obtained using two distinct methods, a "test method" ( $X$ ) and a "reference method" ( $Y$ ), estimate their linear association*

## Precisely

*For a given 25(OH)D value  $X$  obtained with the "test method" estimate the corresponding "reference method" value,  $Y$*



# Introduction

## VDSP Statistics Manual

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## Notation

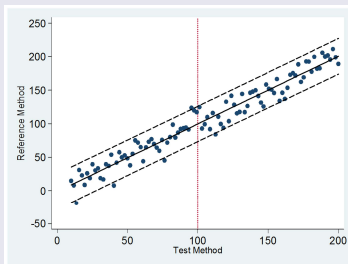
- For the **VDSP**, **Reference Method** stands for Ghent University and NIST reference method procedures.
- **Test Method** relates to the measurement procedure used to obtain 25(OH)D values in "past" studies.
- The units of measurement through this presentation are **nmol/L**

# The Measurement Model

## Linear Regression

$$Y = \beta_0 + \beta_1 X + \epsilon$$

$$\epsilon \sim N(0, \sigma^2)$$



## General Error in Regression

$$Y = \tilde{Y} + \epsilon_y$$

$$X = \tilde{X} + \epsilon_x$$

$$\tilde{Y} = \beta_0 + \beta_1 \tilde{X} + \epsilon$$

$$\epsilon_y \sim N(0, \sigma_y^2)$$

$$\epsilon_x \sim N(0, \sigma_x^2)$$

$$\epsilon \sim N(0, \sigma^2)$$



# Linear Regression

Test Method is error-free

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An estimate of:

$$\beta_0 + \beta_1 X$$

is given by

$$a + bX,$$

where **a**, and **b** estimate the intercept and slope, respectively.

# Estimation

Naive Case: 95% CI for  $\beta_0 + \beta_1 X$

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$$(a+bX) \pm 1.96 \times \sigma_{y \cdot x} \sqrt{\frac{1}{N} + \frac{(X - \bar{X})^2}{(N-1)S_x^2}}$$

- $\bar{X}$  mean of test values
- $S_x^2$  variance of the test values.
- The error of the reference method for a given X value is:

$$\sigma_{y \cdot x}$$

Precision (width) depends on:

- The distribution of the test method values
  - Value of X and its distance from the mean ( $\bar{X}$ )
  - Standard deviation of X values ( $S_x$ )
- The error of the reference method
- Sample size, **N**



# Sample Selection Algorithm

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- Random sampling
- Uniform random sampling
- Uniform random sampling in each of the 4 quartiles of the distribution





# Sample Selection

Uniform vs Uniform quartile selection

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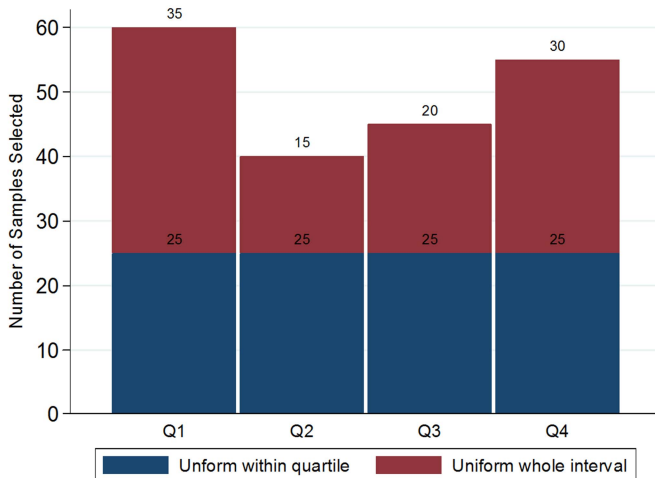
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# Sample Selection Algorithm

Example: Irish National Adult Nutrition Survey

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- Estimate the required sample size, say  **$N=100$**
- Select  **$k$**  samples in each of the 4 quartiles,  **$k = N/4$**
- Estimate quartiles of distribution:  **$42, 57, 74$**  (nmol/L)
- For each interval,  **$[A,B]$** , compute  **$D = (B - A)$**  and divide by  **$k$**  (the number of samples to be selected from each of the intervals).
  - $[20 \text{ nmol/L}, 42 \text{ nmol/L}), D = 22, \frac{D}{25} = 0.88$
  - $[42 \text{ nmol/L}, 57 \text{ nmol/L}), D = 15, \frac{D}{25} = 0.60$
  - $[57 \text{ nmol/L}, 74 \text{ nmol/L}), D = 17, \frac{D}{25} = 0.68$
  - $[74 \text{ nmol/L}, 200 \text{ nmol/L}), D = 126, \frac{D}{25} = 5.04$



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Example: Irish National Adult Nutrition Survey

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- For each interval,  $[A, B]$ , sort the samples from smallest to largest
- In each of the intervals, select the  $k$  values by sequentially adding  $D/k$  to  $A$ . For example for the the interval from **20 nmol/L** to **42 nmol/L**,  $D/k = 0.88$ . The  $k=25$  values are:

$20, 20.88, 21.76, \dots, 40.24, 41.12, \text{ nmol/L}$

- When the target 25(OH)D value does not exist in your sample, select the nearest value
- Record the lot number for the selected samples. Manual adjustment may be necessary if the selected lots appear non-representative



# Sample Selection

Statistics Coordinating Center

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- Prepare the de-identified data including (and only including) the following information
  - Individual ID
  - 25-Hydroxyvitamin D measurement value
  - Lot and date of the measurement
- The study members will keep the link between the individual **ID** sent out and the study sample **ID**, which can be used to identify the actual samples. That information will not be sent to the VDSP statistics coordinating center
- Send the data to the VDSP coordinating center
- Receive individual ID representing selected samples and link the individual ID with actual sample



# Sample Selection

## Example

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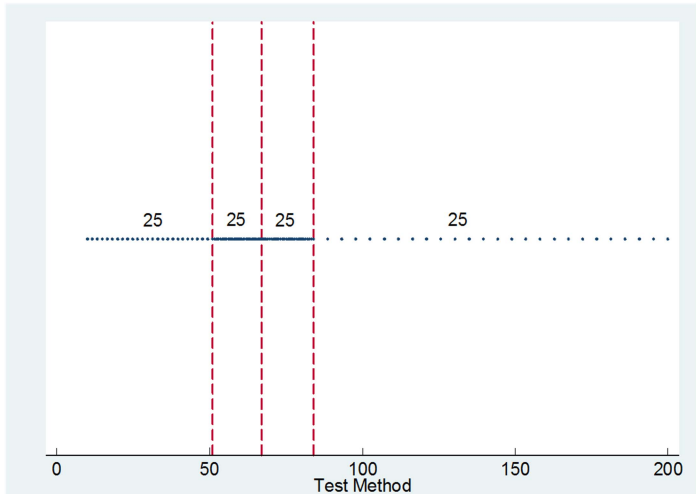
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# Sample Size Estimation

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## Example: Assumptions

- $X$ , the test method value is measured without error
- The error of the reference method,  $Y$ , is independent of  $X$
- Sample will be selected uniformly from  $[20 \text{ nmol/L}, 200 \text{ nmol/L}]$
- The CV% for the reference method at the mean value ( $\bar{X} = 110, \text{ nmol/L}$ ), is known (5%)

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## Example: Assumptions

- Mean X value =  $\bar{X} = 110 \text{ nmol/L}$
- Standard deviation of X values,  $S_x = \frac{180}{\sqrt{12}} = 51.96$
- The CV% for the reference method at the mean value ( $\bar{X} = 110 \text{ nmol/L}$ ), is known (5%)
- Error of reference method:  
$$\sigma_{y \cdot x} = 0.05 \times \bar{X} = 0.05 \times 110 = 5.5 \text{ nmol/L}$$

# Sample Size (N) Estimation

Test method error-free

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## Sample Size (N) Estimation: 95% Prediction CI

### Assumptions:

- Uniform sampling
- Width = 3 nmol/L
- Min = 20 nmol/L, Max = 200 nmol/L
- $\sigma_{y \cdot x} = 5.5 \text{ nmol/L}$
- $CV_y = 5\%$  at 110 nmol/L

| 25(OH)D Value<br>(nmol/L) | Sample Size<br>( $N$ ) |
|---------------------------|------------------------|
| 30                        | 175                    |
| 50                        | 121                    |
| 75                        | 76                     |
| 100                       | 54                     |





# Statistical Analysis

Test method non-error-free

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## General Error in Regression

$$Y = \tilde{Y} + \epsilon_y$$

$$X = \tilde{X} + \epsilon_x$$

$$\tilde{Y} = \beta_0 + \beta_1 \tilde{X} + \epsilon$$

$$\sigma_y = CV_y \times \tilde{Y}$$

$$\sigma_x = CV_x \times \tilde{X}$$

## Estimation Methods

- Deming Regression (Homogeneous Measurement Error)
- Weighted Deming Regression (Heterogeneous Measurement Error)
- Other Methods (Heterogeneous Measurement Error) Lu T, Durazo-Arvizu RA, Myers G, Brooks S, Sarafin K, Sempos TS. *The Estimation of Calibration Equations for Variables with Heteroscedastic Measurement Error*. Under review (Statistics in Medicine)



# Statistical Analysis

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## Sample Size (N) Estimation: 95% Prediction CI

### Assumptions:

- Width = 3 nmol/L
- Min = 20 nmol/L, Max = 200 nmol/L
- Quartiles: 41.6, 57.2, 74.1 nmol/L
- $CV_x = 6.6\%$
- $CV_y = 5\%$

| 25(OH)D Value<br>(nmol/L) | Uniform<br>(N) | Quartile<br>(N) |
|---------------------------|----------------|-----------------|
| 30                        | 231            | 97*             |
| 50                        | 131            | 39              |
| 75                        | 86             | 104             |
| 100                       | 130            | 323             |

\* Irish National Adult Nutrition Survey



# Sample Size (N) Estimation

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## Sample Size (N) Estimation: 95% Prediction CI

### Assumptions:

- Maximum value = 125 nmol/L
- Width = 5 nmol/L
- Uniform Sampling
- $CV_y = 5\%$
- 25(OH)D = 30 nmol/L

| Minimum Value | Coefficient of Variation Test Method |     |           |     |
|---------------|--------------------------------------|-----|-----------|-----|
|               | 5%                                   | 10% | 12%       | 15% |
| 10            | 13                                   | 29  | <b>37</b> | 51  |
| 15            | 17                                   | 40  | <b>52</b> | 72  |
| 20            | 24                                   | 56  | <b>73</b> | 102 |



# Sample Size (N) Estimation

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## Sample Size (N) Estimation: 95% Prediction CI

### Assumptions:

- Maximum value = 150 nmol/L
- Width = 5 nmol/L
- Uniform Sampling
- $CV_y = 5\%$
- 25(OH)D = 30 nmol/L

| Minimum Value | Coefficient of Variation Test Method |     |            |     |
|---------------|--------------------------------------|-----|------------|-----|
|               | 5%                                   | 10% | 12%        | 15% |
| 10            | 20                                   | 46  | <b>60</b>  | 81  |
| 15            | 26                                   | 61  | <b>79</b>  | 108 |
| 20            | 34                                   | 81  | <b>105</b> | 145 |



# Sample Size (N) Estimation

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## Sample Size (N) Estimation: 95% Prediction CI

### Assumptions:

- Maximum value = 175 nmol/L
- Width = 5 nmol/L
- Uniform Sampling
- $CV_y = 5\%$
- 25(OH)D = 30 nmol/L

| Minimum Value | Coefficient of Variation Test Method |     |            |     |
|---------------|--------------------------------------|-----|------------|-----|
|               | 5%                                   | 10% | 12%        | 15% |
| 10            | 30                                   | 68  | <b>88</b>  | 119 |
| 15            | 37                                   | 87  | <b>112</b> | 152 |
| 20            | 47                                   | 110 | <b>142</b> | 195 |



# Sample Size(N) Estimation

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## Sample Size Estimation: 95% Prediction CI

### Assumptions:

- Maximum value = 200 nmol/L
- Width = 5 nmol/L
- Uniform Sampling
- $CV_y = 5\%$
- 25(OH)D = 30 nmol/L

| Minimum Value | Coefficient of Variation Test Method |     |            |     |
|---------------|--------------------------------------|-----|------------|-----|
|               | 5%                                   | 10% | 12%        | 15% |
| 10            | 41                                   | 95  | <b>122</b> | 164 |
| 15            | 50                                   | 117 | <b>150</b> | 204 |
| 20            | 62                                   | 144 | <b>185</b> | 253 |



# Acknowledgments

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# Thank You!

# Questions?